Sample Final Exam Questions

1. (a) State (in words) what

$$\lim_{x \to a} f(x) = L$$

means.

   (b) Define what it means for $f(x)$ to be continuous at a.

   (c) Define the derivative of $f(x)$ at a. (Either limit is acceptable).

   (d) Define the integral of $f(x)$ from a to b.

2. (a) State the Fundamental Theorem of Calculus, Part I.

   (b) State the Fundamental Theorem of Calculus, Part II.

3. Find the value of the following limits:

   (a) $\lim_{x \to \infty} \dfrac{2x^3 - x^2 + 3x - 2}{1 + x + 6x^3}$

   (b) $\lim_{x \to 1} \dfrac{x^{10} - 1}{x^4 - 1}$

   (c) $\lim_{x \to \infty} \dfrac{e^x}{x}$

   (d) $\lim_{x \to 0} \dfrac{1 - \cos x}{\sin x}$

4. Find the derivative of the following functions:

   (a) $f(x) = (e^x + 1)(\sin x + 2)$

   (b) $f(x) = \dfrac{e^{2x}}{x^2 + 1}$

   (c) $f(x) = \displaystyle\int_5^x x^3 - x^2 \, dx.$

5. Find the equation of the normal line to $y = e^{x^2+3}$ at $x = 0$.

6. Find the derivative of y with respect to x for the curve

$$y^3 - \sin y + 3 = \frac{x}{x^2 + 1}$$

7. The volume of a cube is increasing at a rate of $10\text{cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30cm?

8. A waterskier, moving at a speed of 30ft/s, skis over a ramp with height 4ft and length 15ft. How fast is she rising as she leaves the ramp?

9. Find the maximum and mininum values of f on the given interval.

   (a) $f(x) = x^4 - 2x^2 + 3$ on $[-1, 4]$.

   (b) $f(x) = \dfrac{x^2 - 4}{x^2 + 4}$ on $[-4, 4]$.

   (c) $f(x) = xe^{-x}$ on $[0, 2]$.

   (d) $f(x) = \sin x + \cos x$ on $[0, \frac{\pi}{3}]$.

10. For the following curves, find the intervals of increase/decrease, the max/min points, intervals of concavity, and inflection points. Then sketch the graph.

    (a) $y = 2 - 2x - x^3$

    (b) $y = \dfrac{x^2}{x + 8}$

    (c) $y = e^{2x - x^2}$

    (d) $y = \sin(2x)$ on the interval $[0, \pi]$.

11. Describe the horizontal and vertical asympotes of the following curve:

$$y = \frac{x^2 - 1}{3x^2 + 6x - 24}$$

12. Find the Riemann sum of $f(x) = x^3 - 2$ from 1 to 5 using $n = 4$.

13. Find the general antiderivatives of:

    (a) $f(x) = e^x - 6$

    (b) $f(x) = \dfrac{-2}{\sqrt{x}}$

    (c) $f(x) = \dfrac{x^2 + x}{x^3}$

(d) $f(x) = 3\sin x - 2\cos x$

14. Evaluate the following integrals:

(a) $\displaystyle\int_2^8 4x + 3 \; dx$

(b) $\displaystyle\int_{-5}^5 \frac{5}{x^3} \; dx$

(c) $\displaystyle\int (4 - x)^9 \; dx$

(d) $\displaystyle\int \frac{x}{(x^2 + 1)^2} \; dx$

(e) $\displaystyle\int \cos\theta \sin^6\theta \; d\theta$

(f) $\displaystyle\int_0^1 \frac{e^x}{e^x + 1} \; dx$

15. Find the area under the curve $y = x^3 - 2x^2 + 3$ from $x = 2$ to $x = 3$.